P2 Chapter 8 Parametric Equations

Chapter Overview

This chapter is very similar to the trigonometry chapters in Year 1. The only difference is that new trig functions: sec, $cosec$ and cot, are introduced.

1:: Converting from parametric to Cartesian form.

If $x = 2\cos t + 1$ and $y = 3\sin t$, find a Cartesian equations connecting x and y.

2:: Sketching parametric curves.

Sketch the curve with parametric equations $x = 2t$ and $y = \frac{5}{t}$.

3:: Finding points of intersection.

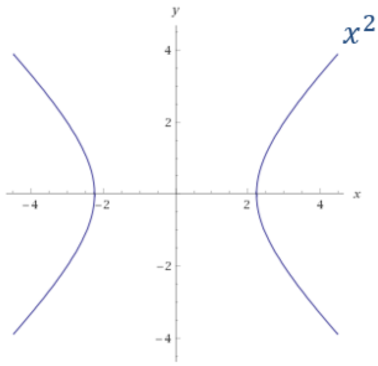
Curve C_1 has the parametric equations $x = t^2$ and $y = 4t$. The curve C_2 has the Cartesian equation $x + y + 4 = 0$. The two curves intersect at A. Find the coordinates of A.

4:: Modelling

A plane's position at time t seconds after take-off can be modelled with the parametric equations:
$x = (v\cos\theta)t$ m, $y = (v\sin\theta)t$ m, $t > 0$
...

Topics	What students need to learn:		
	Content		Guidance
3 **Coordinate geometry in the (x, y) plane** *continued*	3.3	Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.	For example: $x = 3\cos t,\ y = 3\sin t$ describes a circle centre O radius 3 $x = 2 + 5\cos t,\ y = -4 + 5\sin t$ describes a circle centre $(2, -4)$ with radius 5 $x = 5t,\ y = \frac{5}{t}$ 5describes the curve $xy = 25$ (or $y = \frac{25}{x}$) $x = 5t, y = 3t^2$ describes the quadratic curve $25y = 3x^2$ and other familiar curves covered in the specification. Students should pay particular attention to the domain of the parameter t, as a specific section of a curve may be described.
	3.4	Use parametric equations in modelling in a variety of contexts.	A shape may be modelled using parametric equations or students may be asked to find parametric equations for a motion. For example, an object moves with constant velocity from $(1, 8)$ at $t = 0$ to $(6, 20)$ at $t = 5$. This may also be tested in Paper 3, section 7 (kinematics).

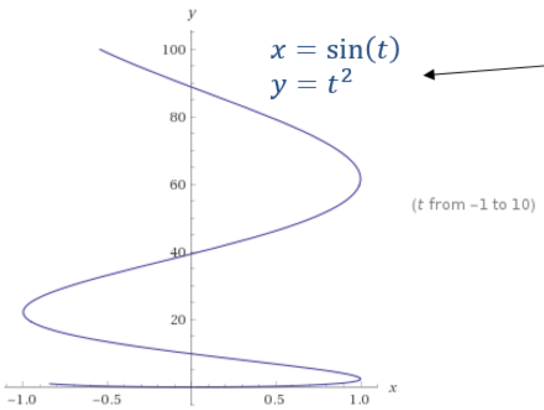
What are they and what is the point?



$x^2 - y^2 = 5$

Typically, with two variables x and y, we can relate the two by **a single equation involving just x and y**.
This is known as a **Cartesian equation**.

The line shows all points (x, y) which satisfy the Cartesian equation.



However, in Mechanics for example, we might want each of the x and y values to be some function of time t, as per this example.

This would allow us to express the position of a particle at time t as the vector:

$$\begin{pmatrix} \sin t \\ t^2 \end{pmatrix}$$

These are known as **parametric equations**, because **each of x and y are defined in terms of some other variable, known as the parameter** (in this case t).

Converting parametric to Cartesian

How could we convert these parametric equations into a single Cartesian one?

$$x = 2t, \qquad y = t^2, \qquad -3 < t < 3$$

What is the domain of the function?

✏ **If $x = p(t)$ and $y = q(t)$ can be written as $y = f(x)$, then the domain of f is the range of p...**

✏ **and the range of f is the range of q.**

Further Example

[Textbook] A curve has the parameter equations

$$x = \ln(t+3), \qquad y = \frac{1}{t+5}, \qquad t > -2$$

a) Find a Cartesian equation of the curve of the form $y = f(x)$, $x > k$, where k is a constant to be found.
b) Write down the range of $f(x)$.

A common strategy for domain/range questions is to consider what happens are the boundary value (in this case -2), then since $t > -2$, consider what happens as t increases.

Test Your Understanding

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The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

(c) Find a cartesian equation of the curve C, in the form $y = f(x)$. **(4)**

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6. The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0.$$

(b) a cartesian equation of C.

(3)

Exercise 8A Page 200