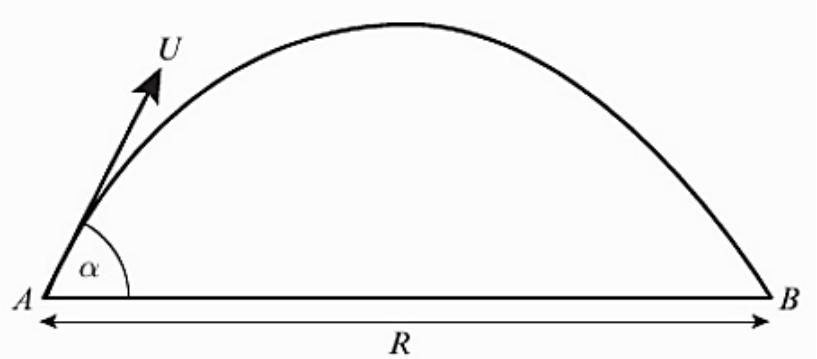
Projection motion Formulae

You must be able to derive general formulae related to the motion of a particle which is projected from a point on a horizontal plane and moves freely under gravity.

Deriving the Time of Flight (T) and the Range (R)



A particle is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal and moves freely under gravity until it hits the plane at point B.

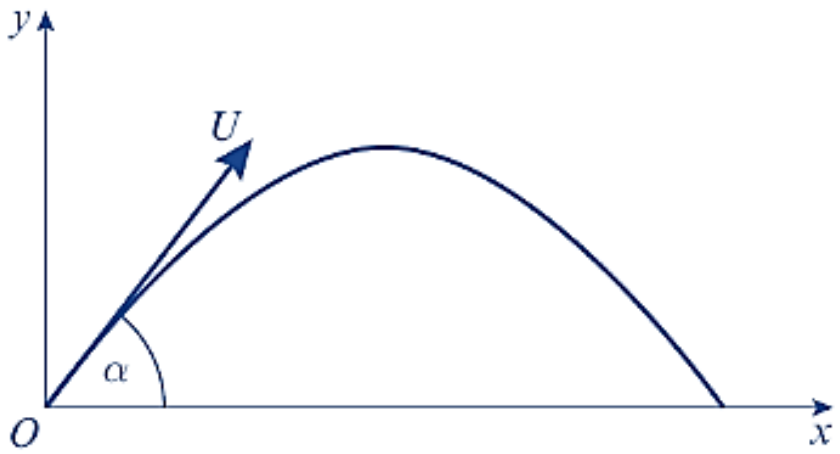
Given that that acceleration due to gravity is g, find expressions for:

(a) the time of flight, T

(b) the range, R, on the horizontal plane.

Deriving the Equation of the Trajectory

When a particle is projected from a point O, on a horizontal plane, the equation of the trajectory may be obtained by taking x and y axes through the point of projection, O, as shown on the diagram.



A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x\tan\alpha\ -\frac{gx^2}{2u^2}(1+\tan^2\alpha)$

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from O and at a height of 8m above the plane.

(b) Find the two possible values of α, giving your answers to the nearest degree.

Exam Note: You may be asked to derive these. But don't attempt to memorise them or actually use them to solve exam problems – instead use the techniques used earlier in the chapter.

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \frac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \frac{U \sin \alpha}{g}$
- Range on horizontal plane $= \frac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory: $y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$
  where y is vertical height of particle and x horizontal distance.

Exercise 6D Page 123
Mixed Exercise 6 Page 125