**Projection motion Formulae**

You must be able to derive general formulae related to the motion of a particle which is projected from a point on a horizontal plane and moves freely under gravity.

**Deriving the Time of Flight (T) and the Range (R)**



A particle is projected from a point on a horizontal plane with an initial velocity $U$ at an angle $α$ above the horizontal and moves freely under gravity until it hits the plane at point $B$.

Given that that acceleration due to gravity is $g$, find expressions for:

1. the time of flight, $T$
2. the range, $R$, on the horizontal plane.

**Deriving the Equation of the Trajectory**

When a particle is projected from a point O, on a horizontal plane, the equation of the trajectory may be obtained by taking x and y axes through the point of projection, O, as shown on the diagram.



A particle is projected from a point with speed $U$ at an angle of elevation $α$ and moves freely under gravity. When the particle has moved a horizontal distance $x$, its height above the point of projection is $y$.

1. Show that $y=x\tan(α -\frac{gx^{2}}{2u^{2}}\left(1+tan^{2}α\right))$

A particle is projected from a point $O$ on a horizontal plane, with speed 28 ms-1 at an angle of elevation $α$. The particle passes through a point $B$, which is at a horizontal distance of 32m from $O$ and at a height of 8m above the plane.

(b) Find the two possible values of $α$, giving your answers to the nearest degree.

**Exam Note**: You may be asked to derive these. But don’t attempt to memorise them or actually use them to solve exam problems – instead use the techniques used earlier in the chapter.

For a particle projected with initial velocity $U$ at angle $α$ above horizontal and moving freely under gravity:

* Time of flight $=\frac{2U\sin(α)}{g}$
* Time to reach greatest height $=\frac{U\sin(α)}{g}$
* Range on horizontal plane $=\frac{U^{2}\sin(2α)}{g}$
* Equation of trajectory: $y=x\tan(α -\frac{gx^{2}}{2U^{2}}\left(1+tan^{2}α\right))$
where $y$ is vertical height of particle and $x$ horizontal distance.

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