

Projection at Any Angle

We can solve problems with particles projected at any angle by resolving the initial velocity into horizontal and vertical components.

Range = distance from point at which the particle was projected to the point where it strikes the horizontal plane

Time of Flight = time taken by particle to move from its point of projection to the point where it strikes the horizontal plane

A projectile reaches its point of greatest height when the vertical component of its velocity, $u_y = 0$.

Example

[Textbook] A particle P is projected from a point O on a horizontal plane with speed 28 ms^{-1} and with angle of elevation 30° . After projection, the particle moves freely under gravity until it strikes the plane at a point A . Find:

- (a) the greatest height above the plane reached by P
- (b) the time of flight of P
- (c) the distance OA

Example

[Textbook] A particle is projected from a point O with speed $V \text{ ms}^{-1}$ and at an angle of elevation of θ , where $\tan \theta = \frac{4}{3}$. The point O is 42.5m above a horizontal plane. The particle strikes the plane at a point A , 5 s after it is projected.

(a) Show that $V = 20$. (b) Find the distance between O and A .

Example

[Textbook] A particle is projected from a point O with speed 35 ms^{-1} and at an angle of elevation of 30° . The particle moves freely under gravity. Find the length of time for which the particle is 15 m or more above O .

Test Your Understanding (EdExcel M2 May 2012 Q7)

A small stone is projected from a point O at the top of a vertical cliff OA . The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B , where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1} . (3)

(b) Find the speed of projection. (9)

(c) Find the time after projection when the stone is moving parallel to OB . (5)

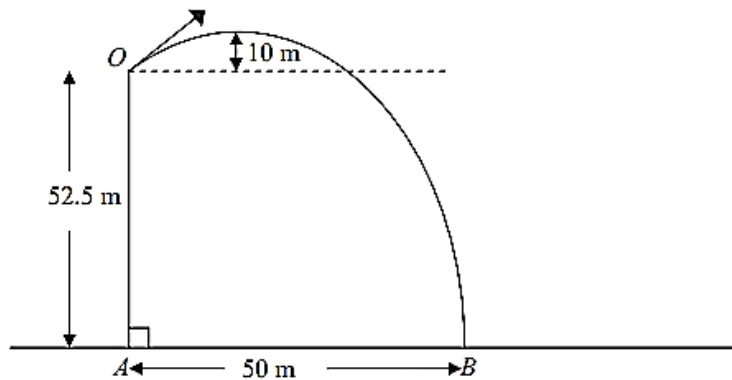


Figure 4

Extension Question:

A ball is projected from ground level at an angle of θ . Prove that when the ball hits the ground, the distance the ball has travelled along the ground is maximised when $\theta = 45^\circ$.
(Year 2 differentiation knowledge required)