Recurrence Relations

Example

6. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$
$$x_{n+1} = (x_n)^2 - kx_n, \qquad n \geq 1,$$

where k is a constant.

(*a*) Find an expression for x_2 in terms of k.

(1)

(*b*) Show that $x_3 = 1 - 3k + 2k^2$.

(2)

Given also that $x_3 = 1$,

(*c*) calculate the value of k.

(3)

(*d*) Hence find the value of $\sum_{n=1}^{100} x_n$.

(3)

Test Your Understanding

4. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = a x_n + 5, \qquad n \geq 1,$$

where a is a constant.

(*a*) Write down an expression for x_2 in terms of a.

(1)

(*b*) Show that $x_3 = a^2 + 5a + 5$.

(2)

Given that $x_3 = 41$

(*c*) find the possible values of a.

(3)

Combined Sequences

Sequences (or series) can be generated from a combination of both an arithmetic and a geometric sequence.

Example

4. (i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$ **(4)**

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$ **(3)**

Extension

1. *[AEA 2011 Q3]* A sequence $\{u_n\}$ is given by

$$u_1 = k u_{2n} = u_{2n-1} \times p, \quad n \geq 1 u_{2n+1} = u_{2n} \times q \qquad n \geq 1$$

(a) Write down the first 6 terms in the sequence.

(b) Show that $\sum_{r=1}^{2n} u_r = \frac{k(1+p)(1-(pq)^n)}{1-pq}$

$[x]$ means the integer part of x, for example $[2.73] = 2,\ [4] = 4.$

Find $\sum_{r=1}^{\infty} 6 \times \left(\frac{4}{3}\right)^{\left[\frac{r}{2}\right]} \times \left(\frac{3}{5}\right)^{\left[\frac{r-1}{2}\right]}$

2. [MAT 2014 1H] The function $F(n)$ is defined for all positive integers as follows: $F(1) = 0$ and for all $n \geq 2$,

$F(n) = F(n-1) + 2$ if 2 divides n but 3 does not divide ,

$F(n) = F(n-1) + 3$ if 3 divides n but 2 does not divide n,

$F(n) = F(n-1) + 4$ if 2 and 3 both divide n

$F(n) = F(n-1)$ if neither 2 nor 3 divides n.

Then the value of $F(6000)$ equals what?

3. [MAT 2016 1G] The sequence (x_n), where $n \geq 0$, is defined by $x_0 = 1$ and

$x_n = \sum_{k=0}^{n-1}(x_k)$ for $n \geq 1$

Determine the value of the sum $\sum_{k=0}^{\infty} \frac{1}{x_k}$