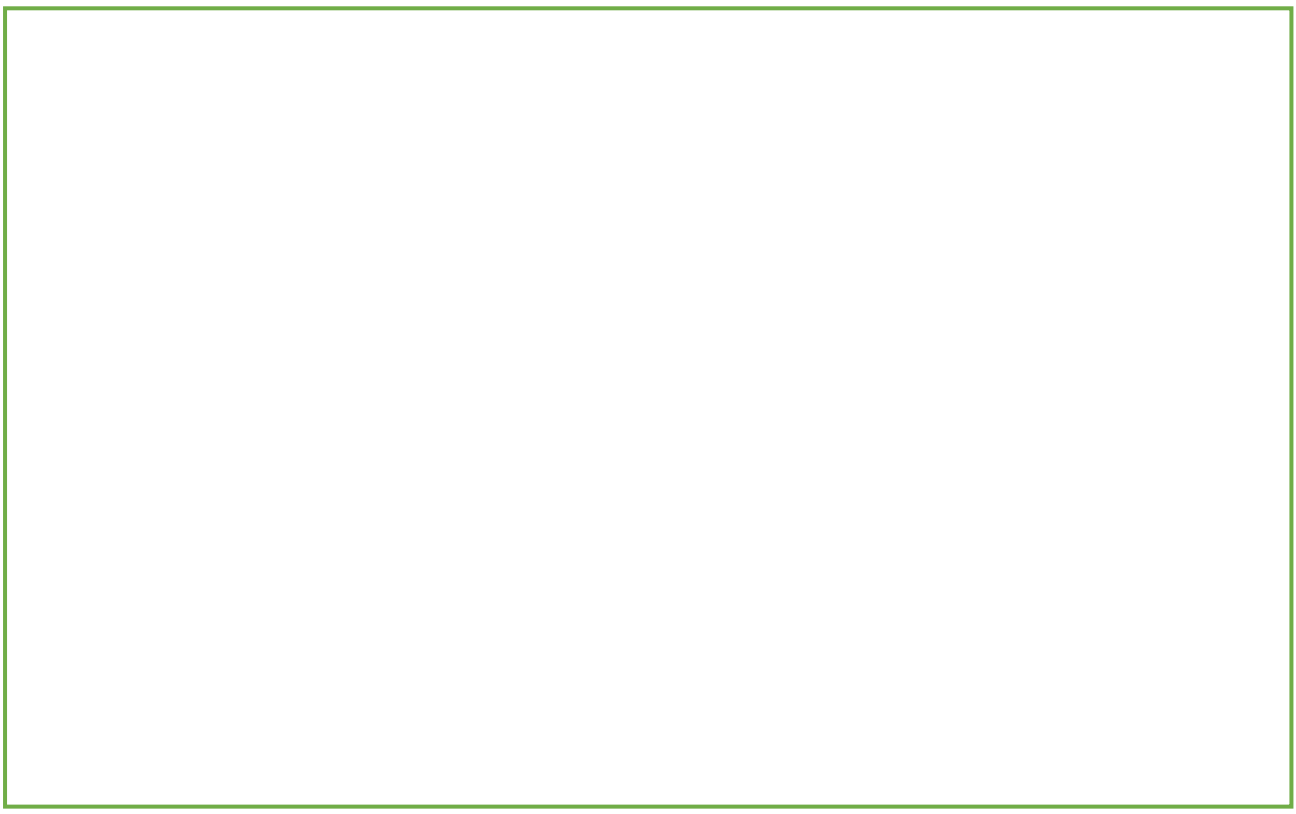
Divergence and Convergence

Sum to Infinity



Quickfire Examples: Calculate a, r and $\boldsymbol{S}_\infty$ for the following sequences

1. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

2. $27, -9, 3, -1, \ldots$

3. $p,\ p^2, p^3, p^4, \ldots \quad where\ -1 < p < 1$

4. $p,\ 1, \frac{1}{p}, \frac{1}{p^2}, \ldots$

Examples

1. The fourth term of a geometric series is 1.08 and the seventh term is 0.23328.

   a) Show that this series is convergent.

   b) Find the sum to infinity of this series.

2. For a geometric series with first term a and common ratio r, $S_4 = 15$ and $S_\infty = 16$.

a) Find the possible values of r.

b) Given that all the terms in the series are positive, find the value of a.

Test Your Understanding

6. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio,

(2)

(*b*) the first term,

(2)

(*c*) the sum to infinity,

(2)

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

Extension

1. [MAT 2006 1H] How many solutions does the equation

$$2 = \sin x + \sin^2 x + \sin^3 x + \sin^4 x + \cdots$$

have in the range $0 \leq x < 2\pi$

2. [MAT 2003 1F] Two players take turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

3. [Frost] Determine the value of x where:

$$x = \frac{1}{1} + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \frac{6}{32} + \cdots$$

(Hint: Use an approach similar to proof of geometric S_n formula)