## Arithmetic Series

## Proof of summation (required for exam):

Examples

1. Find the sum of the first 30 terms of the following arithmetic sequences

$2 + 5 + 8 + 11 + 14 \ldots$

$100 + 98 + 96 + \cdots$

$p + 2p + 3p + \cdots$

2. Find the greatest number of terms for the sum of $4 + 9 + 14 + \cdots$ to exceed 2000

## Test Your Understanding

**9.** A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £$P$.
Salary increases by £$(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £$(P + 1800)$.
Salary increases by £$T$ each year, forming an arithmetic sequence.

($a$) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T).$$

**(2)**

For the 10-year period, the total earned is the same for both salary schemes.

($b$) Find the value of $T$.

**(4)**

For this value of $T$, the salary in Year 10 under Salary Scheme 2 is £29 850.

($c$) Find the value of $P$.

**(3)**

## Extension

[MAT 2007 1J]
The inequality

$$(n+1)+(n^4+2)+(n^9+3)+\cdots \quad +(n^{10000}+100) > k$$

Is true for all $n \geq 1$. It follows that

A) $k < 1300$
B) $k^2 < 101$
C) $k \geq 101^{10000}$
D) $k < 5150$

[AEA 2010 Q2]
The sum of the first $p$ terms of an arithmetic series is $q$ and the sum of the first $q$ terms of the same arithmetic series is $p$, where $p$ and $q$ are positive integers and $p \neq q$.
Giving simplified answers in terms of $p$ and $q$, find

a) The common difference of the terms in this series,
b) The first term of the series,
c) The sum of the first $(p+q)$ terms of the series.

[MAT 2008 1I]
The function $S(n)$ is defined for positive integers $n$ by

$S(n) =$ sum of digits of $n$

For example, $S(723) = 7+2+3 = 12$.
The sum

$S(1)+S(2)+S(3)+\cdots+S(99)$

equals what?