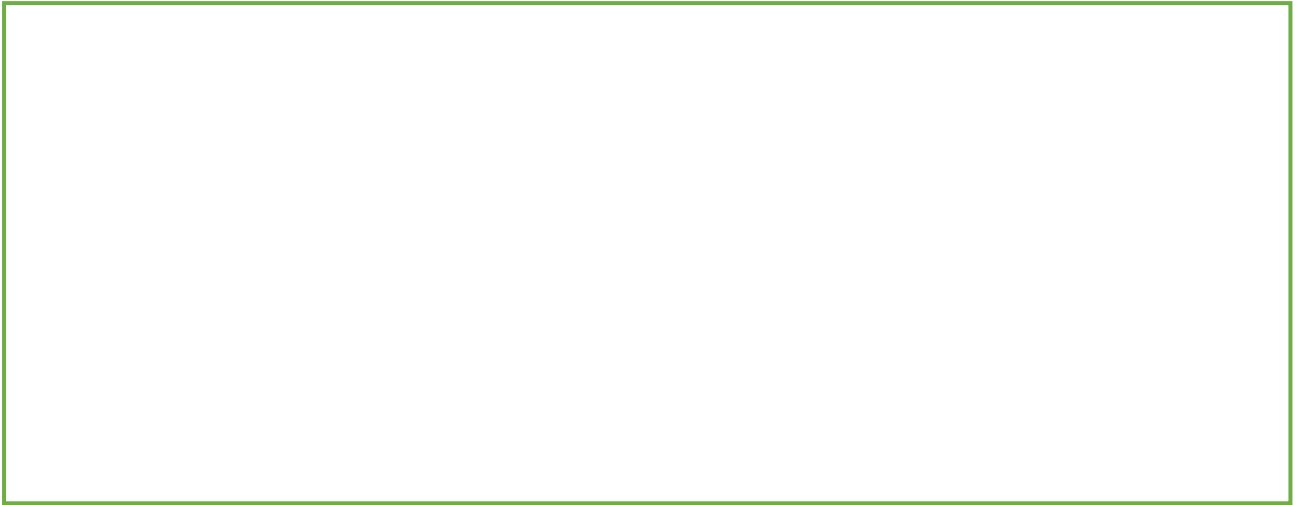


Arithmetic Series



Proof of summation (required for exam):

Examples

1. Find the sum of the first 30 terms of the following arithmetic sequences

$$2 + 5 + 8 + 11 + 14 \dots$$

$$100 + 98 + 96 + \dots$$

$$p + 2p + 3p + \dots$$

2. Find the greatest number of terms for the sum of $4 + 9 + 14 + \dots$ to exceed 2000

Test Your Understanding

9. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is $\pounds P$.

Salary increases by $\pounds(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is $\pounds(P + 1800)$.

Salary increases by $\pounds T$ each year, forming an arithmetic sequence.

- (a) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$\pounds(10P + 90T). \quad (2)$$

For the 10-year period, the total earned is the same for both salary schemes.

- (b) Find the value of T . (4)

For this value of T , the salary in Year 10 under Salary Scheme 2 is $\pounds 29\,850$.

- (c) Find the value of P . (3)

Extension

[MAT 2007 1J]

The inequality

$$(n + 1) + (n^4 + 2) + (n^9 + 3) + \dots + (n^{10000} + 100) > k$$

Is true for all $n \geq 1$. It follows that

- A) $k < 1300$
- B) $k^2 < 101$
- C) $k \geq 101^{10000}$
- D) $k < 5150$

[AEA 2010 Q2]

The sum of the first p terms of an arithmetic series is q and the sum of the first q terms of the same arithmetic series is p , where p and q are positive integers and $p \neq q$.

Giving simplified answers in terms of p and q , find

- a) The common difference of the terms in this series,
- b) The first term of the series,
- c) The sum of the first $(p + q)$ terms of the series.

[MAT 2008 1I]

The function $S(n)$ is defined for positive integers n by

$$S(n) = \text{sum of digits of } n$$

For example, $S(723) = 7 + 2 + 3 = 12$.

The sum

$$S(1) + S(2) + S(3) + \dots + S(99)$$

equals what?