U6 Chapter 3

Sequences and Series

Chapter Overview

1. Sequences
2. Arithmetic Series
3. Geometric Series
4. Sigma Notation
5. Recurrence Relations
6. Combined Sequences
7. Classifying Sequences

4 **Sequences and series** *continued*	4.2	Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$; increasing sequences; decreasing sequences; periodic sequences.	For example $u_n = \dfrac{1}{3n+1}$ describes a decreasing sequence as $u_{n+1} < u_n$ for all integer n $u_n = 2^n$ is an increasing sequence as $u_{n+1} > u_n$ for all integer n $u_{n+1} = \dfrac{1}{u_n}$ for $n > 1$ and $u_1 = 3$ describes a periodic sequence of order 2
	4.3	Understand and use sigma notation for sums of series.	Knowledge that $\sum_{1}^{n} 1 = n$ is expected
	4.4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms	The proof of the sum formula for an arithmetic sequence should be known including the formula for the sum of the first n natural numbers.
	4.5	Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$; modulus notation	The proof of the sum formula should be known. Given the sum of a series students should be able to use logs to find the value of n. The sum to infinity may be expressed as S_∞
	4.6	Use sequences and series in modelling.	Examples could include amounts paid into saving schemes, increasing by the same amount (arithmetic) or by the same percentage (geometric) or could include other series defined by a formula or a relation.

Sequences

A sequence is an ordered set of mathematical objects. Each element in the sequence is called a term.

$u_n =$

$n =$

Arithmetic Sequences

Examples

1. The nth term of an arithmetic sequence is $u_n = 55 - 2n$.

   a) Write down the first 3 terms of the sequence.

   b) Find the first term in the sequence that is negative.

2. Find the nth term of each arithmetic sequence.

a) 6, 20, 34, 48, 62

b) 101, 94, 87, 80, 73

3. A sequence is generated by the formula $u_n = an + b$ where a and b are constants to be found. Given that $u_3 = 5$ and $u_8 = 20$, find the values of the constants a and b.

4. For which values of x would the expression $-8,\ x^2$ and $17x$ form the first three terms of an arithmetic sequence.

Test Your Understanding

Xin has been given a 14 day training schedule by her coach.

Xin will run for A minutes on day 1, where A is a constant.

She will then increase her running time by $(d + 1)$ minutes each day, where d is a constant.

(a) Show that on day 14, Xin will run for

$$(A + 13d + 13) \text{ minutes.}$$

(2)

Yi has also been given a 14 day training schedule by her coach.

Yi will run for $(A - 13)$ minutes on day 1.

She will then increase her running time by $(2d - 1)$ minutes each day.

Given that Yi and Xin will run for the same length of time on day 14,

(b) find the value of d.

(3)

Extension

[STEP I 2004 Q5] The positive integers can be split into five distinct arithmetic progressions, as shown:
A: 1, 6, 11, 16, …
B: 2, 7, 12, 17, …
C: 3, 8, 13, 18, …
D: 4, 9, 14, 19, …
E: 5, 10, 15, 20, …
Write down an expression for the value of the general term in each of the five progressions.
Hence prove that the sum of any term in B and any term in C is a term in E.
Prove also that the square of every term in B is a term in D. State and prove a similar claim about the square of every term in C.

i) Prove that there are no positive integers x and y such that $x^2 + 5y = 243723$
ii) Prove also that there are no positive integers x and y such $x^4 + 2y^4 = 26081974$