Negative Areas

Sketch the curve .

Now calculate .

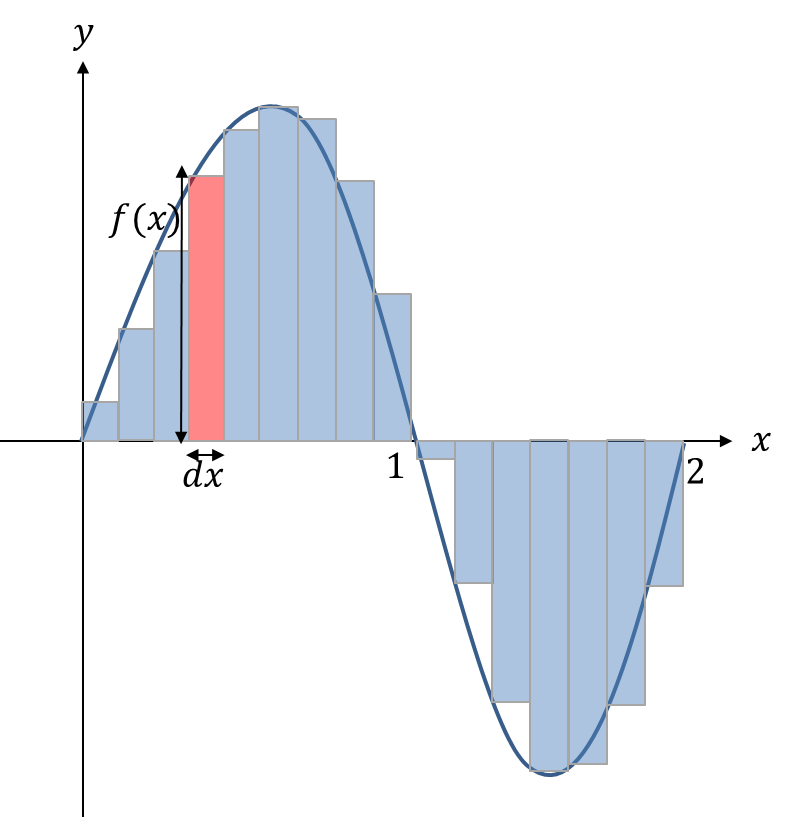
Why is this result surprising?

Integration is just the sum of areas of infinitely thin rectangles, where the current value (i.e. ) is each height, and the widths are .

i.e. The area of each is

The problem is, when is negative, then is negative, i.e. a negative area!

The result is that the ‘positive area’ from 0 to 1 is cancelled out by the ‘negative area’ from 1 to 2, giving an overall ‘area’ of 0.

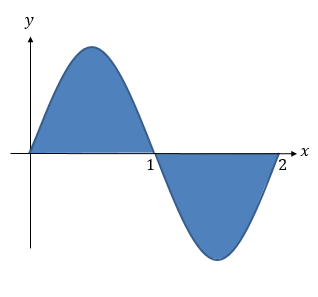


So how do we resolve this?

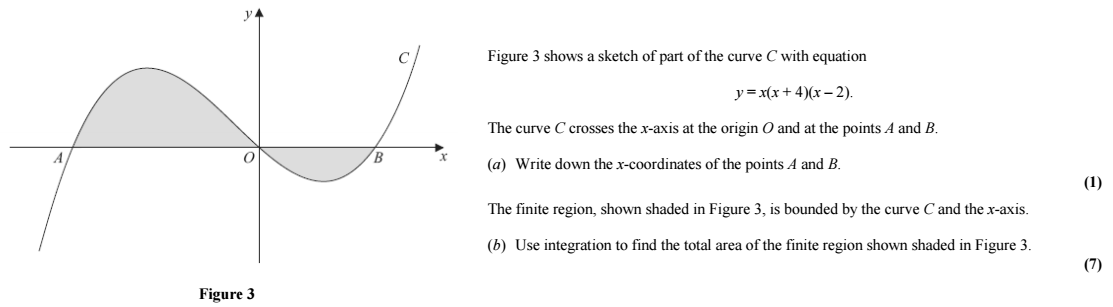
This explains the in the , which effectively means “the sum of the areas of strips, each of area . So the is not just part of the notation, it’s behaving as a physical quantity! (i.e. length

Example

Find the total area bound between the curve and the -axis.



Test Your Understanding



Extension

*[MAT 2010 1I]* For a positive number , let

Then when is what value?

*[STEP I 2014 Q3]*

The numbers and , where , are such that

1. In the case and , find the value of .
2. In the case , show that satisfies  
   Show further, with the help of a sketch, that there is only one (real) value of that satisfies the equation and that it lies between 2 and 3.
3. Show that , where and , and express in terms of . Deduce that

Ex 13F pg 301