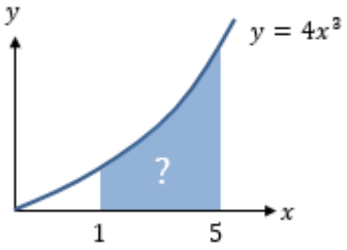
Areas Under Curves

Consider our previous example $\int_1^5 4x^3\ dx$. This definite integral gives the area bounded by the curve and the lines x = 1 and x = 5.



The definite integral $\int_{\boldsymbol{b}}^{\boldsymbol{a}} \boldsymbol{f}(\boldsymbol{x})\ \boldsymbol{dx}$ gives the **area** between a positive curve $y = f(x)$, the $\boldsymbol{x}$**-axis**, and the lines $x = a$ and $x = b$.

Example

Find the area of the finite region between the curve with equation $y = 20 - x - x^2$ and the x-axis.

Test Your Understanding

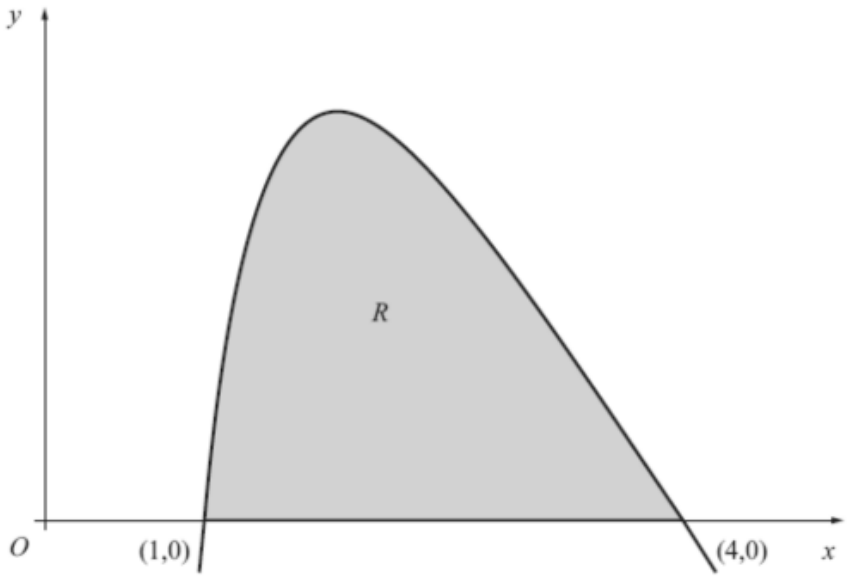


Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \quad x > 0.$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(c) Use integration to find the exact value for the area of R.

(6)

Extension

[MAT 2007 1H] Given a function $f(x)$, you are told that

$$\int_0^1 3f(x)\,dx + \int_1^2 2f(x)\,dx = 7\int_0^2 f(x)\,dx + \int_1^2 f(x)\,dx = 1$$

It follows that $\int_0^2 f(x)\ dx$ equals what?

[MAT 2011 1G]

A graph of the function $y = f(x)$ is sketched on the axes below:

What is the value of $\int_{-1}^{1} f(x^2 - 1)\ dx$?