

Definite Integral

The most useful use of integration is that **it finds the area under a graph**. Before we do this, we need to understand how to find a **definite integral**.

Examples

1. $f(x) = 4x^3$

2. $\int_{-3}^3 x^2 + 1 \, dx =$

3. Given that P is a constant and $\int_1^5 (2Px + 7) \, dx = 4P^2$, show that there are two possible values for P and find these values.

Extension

1. [MAT 2009 1A] The smallest value of

$$I(a) = \int_0^1 (x^2 - a)^2 dx$$

as a varies, is what?

2. [MAT 2015 1D] Let

$$f(x) = \int_0^1 (xt)^2 dt \text{ and } g(x) = \int_0^x t^2 dt$$

Let $A > 0$. Which of the following statements are true?

- A) $g(f(A))$ is always bigger than $f(g(A))$
- B) $f(g(A))$ is always bigger than $g(f(A))$
- C) They are always equal.
- D) $f(g(A))$ is bigger if $A < 1$, and $g(f(A))$ is bigger if $A > 1$.
- E) $g(f(A))$ is bigger if $A < 1$, and $f(g(A))$ is bigger if $A > 1$.