$\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ notation

In 2D you were previously introduced to $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as unit vectors in each of the x and y directions.
It meant for example that $\begin{pmatrix} 8 \\ -2 \end{pmatrix}$ could be written as $8\boldsymbol{i} - 2\boldsymbol{j}$ since $8\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$

Unsurprisingly, **in 3D:**

$$\boldsymbol{i} = \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \ \boldsymbol{j} = \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix}, \ \boldsymbol{k} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{pmatrix}$$

Quickfire Questions

1. Put in i, j, k notation:

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} =$

$\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} =$

2. Write as a column vector:

$4\boldsymbol{j} + \boldsymbol{k} =$

$\boldsymbol{i} - \boldsymbol{j} =$

3. If $A(1,2,3),\ B(4,0,-1)$ then

$\overrightarrow{AB} =$

4. If $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$ then $3\boldsymbol{a} + 2\boldsymbol{b} =$

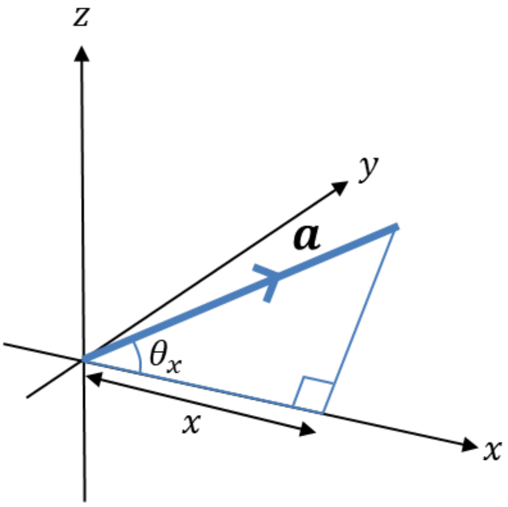
Examples

1. Find the magnitude of $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ and hence find $\hat{\boldsymbol{a}}$, the unit vector in the direction of $\boldsymbol{a}$.

2. If $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$ is $2\boldsymbol{a} - 3\boldsymbol{b}$ parallel to $4\boldsymbol{i} - 5\boldsymbol{k}$?

Angles between vectors and an axis

How could you work out the angle between a vector and the x-axis?



The angle between $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and the x-axis is:

$$\cos \theta_x = \frac{x}{|\boldsymbol{a}|}$$

and similarly for the y and z axes.

[Textbook] **Find the angles that the vector $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$ makes with each of the positive coordinate axis.**

Test Your Understanding

[Textbook] **The points A and B have position vectors $4\boldsymbol{i} + 2\boldsymbol{j} + 7\boldsymbol{k}$ and $3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ relative to a fixed origin, O. Find $\overrightarrow{AB}$ and show that ΔOAB is isosceles.**

(a) Find the angle that the vector $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ makes with the x-axis.

(b) By similarly considering the angle that $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ makes with the x-axis, determine the area of OAB where $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. (Hint: draw a diagram)

Ex 12B p.341-343