Pure 2

Vectors

Chapter Overview

1:: Distance between two points.

2:: i, j, k notation for vectors

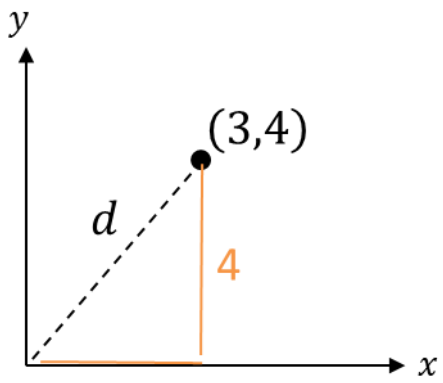
3:: Magnitude of a 3D vector and using it to find angle between vector and a coordinate axis.

4:: Solving Geometric Problems

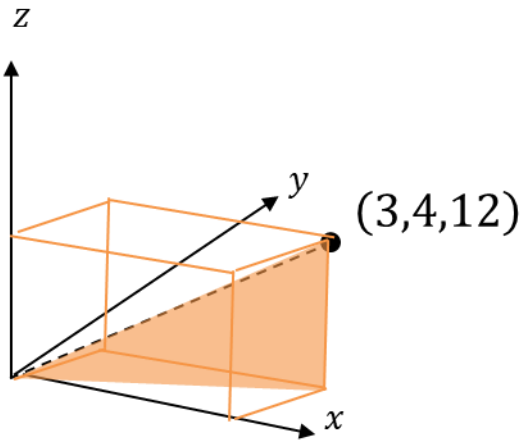
5:: Application to Mechanics

Topics	What students need to learn:		
	Content		Guidance
10 Vectors	10.1	**Use vectors in two dimensions** and in three dimensions	**Students should be familiar with column vectors and with the use of i and j unit vectors in two dimensions** and **i**, **j** and **k** unit vectors in three dimensions.
	10.2	**Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.**	**Students should be able to find a unit vector in the direction of a, and be familiar with the notation $\|a\|$.**
	10.3	**Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.**	**The triangle and parallelogram laws of addition.** **Parallel vectors.**
	10.4	**Understand and use position vectors; calculate the distance between two points represented by position vectors.**	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **The distance d between two points** (x_1, y_1) **and** (x_2, y_2) **is given by** $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$
	10.5	**Use vectors to solve problems in pure mathematics and in context, (including forces).**	**For example, finding position vector of the fourth corner of a shape (e.g. parallelogram) $ABCD$ with three given position vectors for the corners A, B and C.** **Or use of ratio theorem to find position vector of a point C dividing AB in a given ratio.** **Contexts such as velocity, displacement, kinematics and forces will be covered in Paper 3, Sections 6.1, 7.3 and 8.1 – 8.4**

Distance from the origin and magnitude of a vector



In 2D, how did we find the distance from a point to the origin?



How about in 3D then?

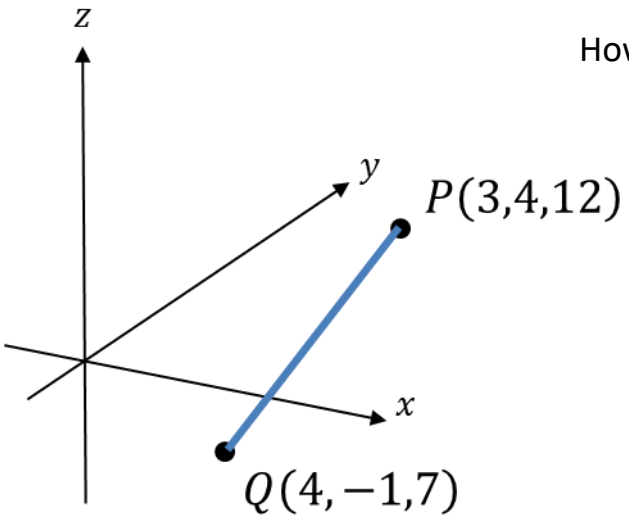
The magnitude of a vector $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$:

$$|\boldsymbol{a}| = \sqrt{x^2 + y^2 + z^2}$$

And the distance of (x, y, z) from the origin is

$$\sqrt{x^2 + y^2 + z^2}$$

Distance between two 3D points



How do we find the distance between P and Q?

The distance between two points is:

$$\boldsymbol{d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}$$

Δx means "change in x"

Quickfire Questions:

Distance of $(4,0,-2)$ from the origin:

$$\left|\begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}\right| =$$

Distance between $(0,4,3)$ and $(5,2,3)$.

Distance between $(1,1,1)$ and $(2,1,0)$.

Distance between $(-5,2,0)$ and $(-2,-3,-3)$.

Tip: Because we're squaring, it doesn't matter whether the change is negative or positive.

Test Your Understanding So Far…

[Textbook] **Find the distance from the origin to the point $P(7,7,7)$.**

[Textbook] **The coordinates of A and B are $(5,3,-8)$ and $(1,k,-3)$ respectively. Given that the distance from A to B is $3\sqrt{10}$ units, find the possible values of k.**

Ex 12A p.338