11K Modelling with Integration

1. The rate of increase of a population P of micro organisms at time t, in hours, is given by:

$$\frac{dP}{dt} = 3P, \quad t > 0$$

Initially, the population was of size 8.

a) Find a model for P in the form $P = Ae^{3t}$, stating the value of A

b) Find, to the nearest hundred, the size of the population at the time $t = 2$

c) Find the time at which the population will be 1000 times its starting value.

d) State one limitation of this model for large values of t

2. Water in a manufacturing plant is held in a large cylindrical tank of diameter 20m. Water flows out of the bottom of the tank through a tap at a rate proportional to the cube root of the volume (of the water).

a) Show that after t minutes after the tap is opened, $\frac{dh}{dt} = -k\sqrt[3]{h}$ for some constant k.

b) Show that the general solution to this differential equation may be written as $h = (P - Qt)^{\frac{3}{2}}$, where P and Q are constants

Initially, the height of the water is 27m. 10 minutes later, the height is 8m.

c) Find the values of the constants P and Q

d) Find the time in minutes when the water is at a depth of 1m