**11K Modelling with Integration**

1. The rate of increase of a population P of micro organisms at time t, in hours, is given by:

$\frac{dP}{dt}=3P, $ $t>0$

Initially, the population was of size 8.

1. Find a model for $P$ in the form $P=Ae^{3t}$, stating the value of $A$
2. Find, to the nearest hundred, the size of the population at the time $t=2$
3. Find the time at which the population will be 1000 times its starting value.
4. State one limitation of this model for large values of $t$
5. Water in a manufacturing plant is held in a large cylindrical tank of diameter 20m. Water flows out of the bottom of the tank through a tap at a rate proportional to the cube root of the volume (of the water).
6. Show that after $t$ minutes after the tap is opened, $\frac{dh}{dt}=-k\sqrt[3]{h}$ for some constant $k$.
7. Show that the general solution to this differential equation may be written as $h=\left(P-Qt\right)^{\frac{3}{2}}$, where $P$ and $Q$ are constants

Initially, the height of the water is 27m. 10 minutes later, the height is 8m.

1. Find the values of the constants $P$ and $Q$
2. Find the time in minutes when the water is at a depth of 1m