Forming differential equations

Differential equations are useful because regularly in real-life, the rate of change of a variable is based on its current value. For example in Year 1, we saw a property of exponential growth is that the **rate of change is proportional to the current value**:

Q The rate of increase of a rabbit population (with population P, where time is t) is **proportional to** the current population.
Form a differential equation, and find its general solution.

Further Example

[Textbook] Water in a manufacturing plant is held in a large cylindrical tank of diameter 20m. Water flows out of the bottom of the tank through a tap at a rate proportional to the cube root of the volume.

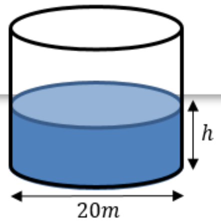
(a) Show that t minutes after the tap is opened, $\frac{dh}{dt} = -k\sqrt[3]{h}$ for some constant k.

(b) Show that the general solution of this differential equation may be written $h = (P - Qt)^{\frac{3}{2}}$, where P and Q are constants.

Initially the height of the water is 27m. 10 minutes later, the height is 8m.

(c) Find the values of the constants P and Q.

(d) Find the time in minutes when the water is at a depth of 1m.



Test Your Understanding

Edexcel C4 June 2005 Q8

Liquid is pouring into a container at a constant rate of 20 $cm^3\ s^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(*a*) Explain why, at time *t* seconds, the volume, *V* cm^3, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where *k* is a positive constant.

(a)

The container is initially empty.

(*b*) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of *A* and *B* in terms of *k*. **(6)**

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(*c*) find the volume of liquid in the container at 10 s after the start. **(5)**

Teachers/Students: I recommend also looking at Edexcel Jan 2008 Q8 which has a part (a) similar to the previous example.