SKILL #11: Differential Equations (We're on the home straight!)

Differential equations are equations involving a mix of variables and derivatives, e.g. y, x and $\frac{dy}{dx}$.

'Solving' these equations means to get y in terms of x (with no $\frac{dy}{dx}$).

Q Find the general solution to $\frac{dy}{dx} = xy + y$

Another Example

Q Find the general solution to $(1 + x^2)\frac{dy}{dx} = x \tan y$

STEP 1: Get y to the side of $\frac{dy}{dx}$ by dividing and x to the other side.
(you may need to factorise to separate out y first)

STEP 2: Integrate both sides with respect to x.

STEP 2b: If possible, try to combine your constant of integration with other terms
(e.g. by letting $C = \ln k$ where k is another constant)

STEP 3: Make y the subject, if the question asks.

Differential Equations with Boundary Conditions

Q [Textbook] Find the general solution to $\frac{dy}{dx} = -\frac{3(y-2)}{(2x+1)(x+2)}$

Given that $x = 1$ when $y = 4$. Leave your answer in the form $y = f(x)$

Test Your Understanding

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Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}.$$ **(5)**

Key Points on Differential Equations

- Get y on to LHS by dividing (possibly factorising first).
- If after integrating you have ln on the RHS, make your constant of integration $\ln k$.
- Be sure to combine all your ln's together just as you did in C2. E.g.:

$$2\ln|x+1| - \ln|x| \quad \rightarrow$$

- Sub in boundary conditions to work out your constant – better to do sooner rather than later.
- Exam questions ♥ partial fractions combined with differential equations.