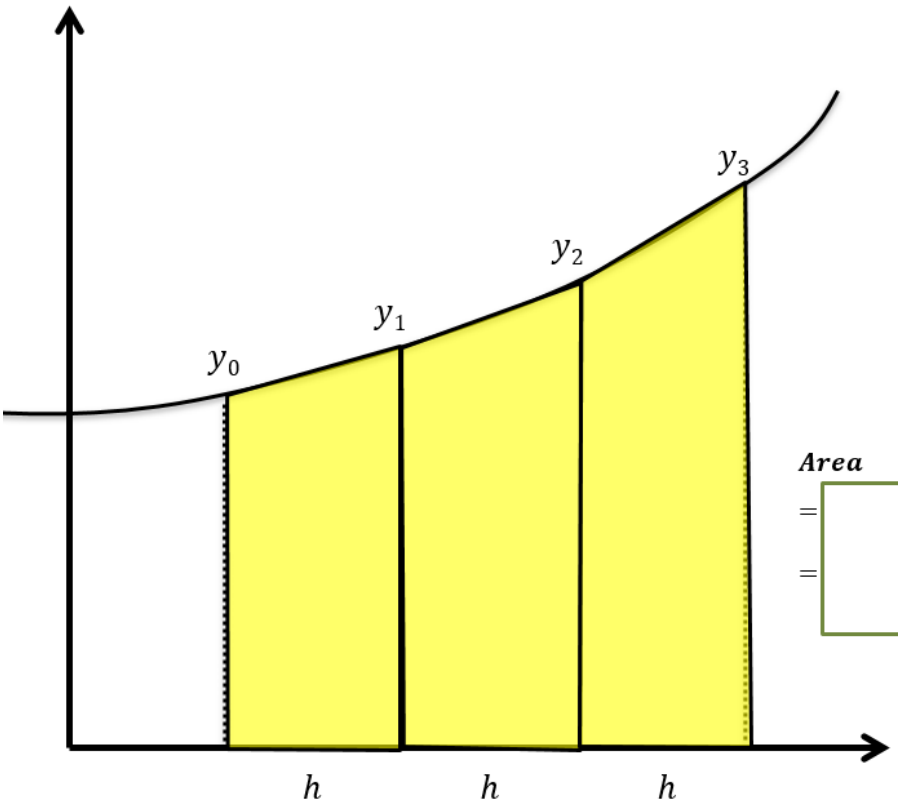
Skill #10: Trapezium Rule



Sometimes finding the exact area under the graph via integration is difficult. Students who have taken GCSE Maths may be familiar with the idea of **approximating the area under a graph by dividing it into trapeziums of equal width**.

$Area$

$=$

$=$

In general:

width of each trapezium

$$\int_a^b y\,dx \approx \frac{h}{2}(y_1 + 2(y_2 + \cdots + y_{n-1}) + y_n)$$

Area under curve

is approximately

Example We're approximating the region bounded between $x = 1$, $x = 3$, the x-axis the curve $y = x^2$, using 4 strips.

Dividing a gap of 2 into 4 strips means each strip will be width 0.5

x	
y	

$h =$

$Area \approx$

Trapezium Rule

Edexcel C2 May 2013 (R) Q2

$$y = \frac{x}{\sqrt{(1+x)}}$$

(*a*) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

Fro Tip: You can generate table with Casio calcs . $Mode \rightarrow 3\ (Table)$. Use 'Alpha' button to key in X within the function. Press =

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\,dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

Area $\approx$

Further Example

Trapezium Rule: $\int_a^b y\,dx \approx \frac{1}{2}h[y_0 + 2(y_1 + \cdots + y_{n-1}) + y_n]$

Q

Given $I = \int_0^{\frac{\pi}{3}} \sec x\ dx$

a) Find the exact value of I.
b) Use the trapezium rule with two strips to estimate I.
c) Use the trapezium rule with four strips to find a second estimate of I.
d) Find the percentage error in using each estimate.

a

b

c

d

Test Your Understanding

Edexcel C4 June 2014(R) Q2

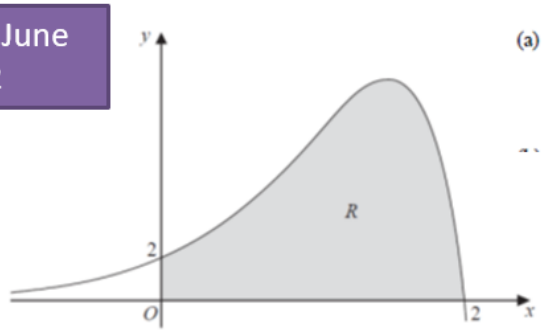


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x},$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$.

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(a) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.

(3)

(b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.

(1)

(c) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.

(5)

(a)

(b)

(c)