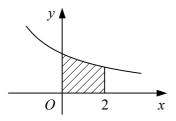
## **C4**

## **INTEGRATION**

## Worksheet G

1



The diagram shows part of the curve with parametric equations

$$x = 2t - 4$$
,  $y = \frac{1}{t}$ .

The shaded region is bounded by the curve, the coordinate axes and the line x = 2.

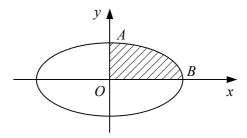
**a** Find the value of the parameter t when x = 0 and when x = 2.

**b** Show that the area of the shaded region is given by  $\int_{2}^{3} \frac{2}{t} dt$ .

**c** Hence, find the area of the shaded region.

**d** Verify your answer to part **c** by first finding a cartesian equation for the curve.

2



The diagram shows the ellipse with parametric equations

$$x = 4 \cos \theta$$
,  $y = 2 \sin \theta$ ,  $0 \le \theta < 2\pi$ ,

which meets the positive coordinate axes at the points A and B.

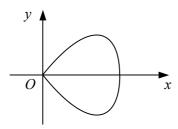
**a** Find the value of the parameter  $\theta$  at the points A and B.

**b** Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by

$$\int_0^{\frac{\pi}{2}} 8 \sin^2 \theta \ d\theta.$$

c Hence, show that the area of the region enclosed by the ellipse is  $8\pi$ .

3



The diagram shows the curve with parametric equations

$$x = 2 \sin t$$
,  $y = 5 \sin 2t$ ,  $0 \le t < \pi$ .

a Show that the area of the region enclosed by the curve is given by  $\int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \, dt$ .

**b** Evaluate this integral.