1 Using integration by parts, show that

$$\int x \cos x \, dx = x \sin x + \cos x + c.$$

2 Use integration by parts to find

$$\mathbf{a} \int x e^x dx$$

b
$$\int 4x \sin x \, dx$$

b
$$\int 4x \sin x \, dx$$
 c $\int x \cos 2x \, dx$

$$\mathbf{d} \quad \int x\sqrt{x+1} \, \mathrm{d}x \qquad \qquad \mathbf{e} \quad \int \frac{x}{e^{3x}} \, \mathrm{d}x$$

$$e \int \frac{x}{e^{3x}} dx$$

$$\mathbf{f} \int x \sec^2 x \, \mathrm{d}x$$

3 Using

i integration by parts,

ii the substitution u = 2x + 1,

find $\int x(2x+1)^3 dx$, and show that your answers are equivalent.

Show that 4

$$\int_0^2 x e^{-x} dx = 1 - 3e^{-2}.$$

5 **Evaluate**

a
$$\int_0^{\frac{\pi}{6}} x \cos x \, dx$$
 b $\int_0^1 x e^{2x} \, dx$

b
$$\int_0^1 xe^{2x} dx$$

$$\mathbf{c} \quad \int_0^{\frac{\pi}{4}} x \sin 3x \, dx$$

Using integration by parts twice in each case, show that 6

a
$$\int x^2 e^x dx = e^x (x^2 - 2x + 2) + c$$
,

b
$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + c.$$

7 Find

$$\mathbf{a} \int x^2 \sin x \, dx$$

$$\mathbf{b} \quad \int x^2 \mathrm{e}^{3x} \, \mathrm{d}x$$

$$\mathbf{c} \int e^{-x} \cos 2x \, dx$$

8 **a** Write down the derivative of $\ln x$ with respect to x.

b Use integration by parts to find

$$\int \ln x \, dx.$$

9 Find

$$\mathbf{a} \int \ln 2x \, dx$$

b
$$\int 3x \ln x \, dx$$

$$\mathbf{c} \int (\ln x)^2 dx$$

Evaluate 10

$$\int_{-1}^{0} (x+2)e^{x} dx$$

$$\mathbf{b} \quad \int_{1}^{2} x^{2} \ln x \, dx$$

a
$$\int_{-1}^{0} (x+2)e^{x} dx$$
 b $\int_{1}^{2} x^{2} \ln x dx$ **c** $\int_{\frac{1}{2}}^{1} 2xe^{3x-1} dx$

d
$$\int_0^3 \ln(2x+3) \, dx$$
 e $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ **f** $\int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx$

$$e \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$$

$$\mathbf{f} \quad \int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx$$