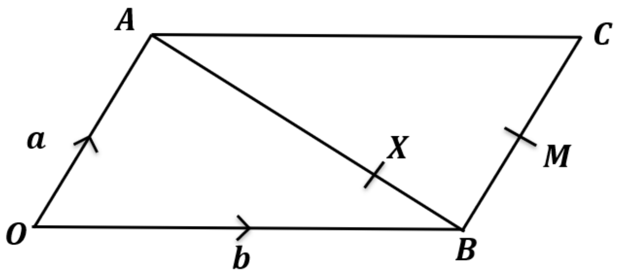
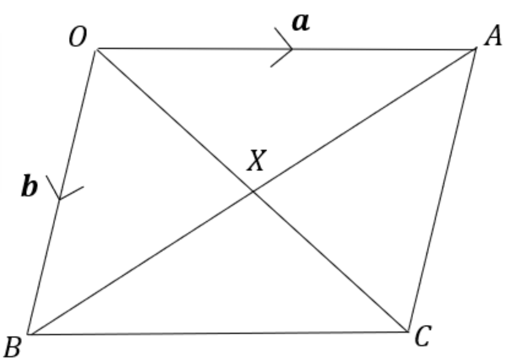
Solving Geometric Problems

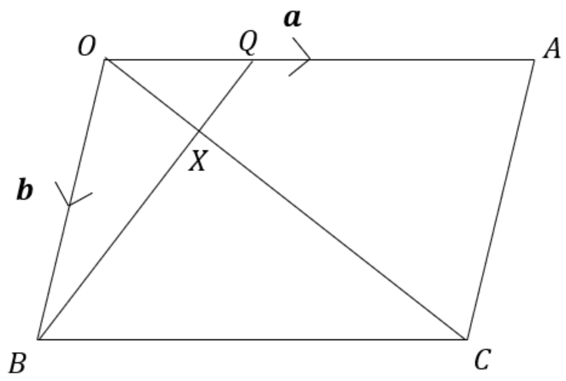


X is a point on AB such that $AX:XB = 3:1$. M is the midpoint of BC.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

$OACB$ is a parallelogram, where $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$. The diagonals OC and AB intersect at a point X. Prove that the diagonals bisect each other.
(Hint: Perhaps find $\overrightarrow{OX}$ in two different ways?)



<u>Test your understanding</u>



In the above diagram, $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OQ} = \frac{1}{3}\boldsymbol{a}$. We wish to find the ratio $OX: XC$.

a) If $\overrightarrow{OX} = \lambda\, \overrightarrow{OC}$, find an expression for $\overrightarrow{OX}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and λ.
b) If $\overrightarrow{BX} = \mu\, \overrightarrow{BQ}$, find an expression for $\overrightarrow{OX}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and μ.
c) By comparing coefficients or otherwise, determine the value of λ, and hence the ratio $OX: XC$.

Area of a triangle example

If $\overrightarrow{AB} = 3\boldsymbol{i} - 2\boldsymbol{j}$ and $\overrightarrow{AC} = \boldsymbol{i} - 5\boldsymbol{j}$. Determine $\angle BAC$.

Extension

[STEP 2010 Q7]

Relative to a fixed origin O, the points A and B have position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively. (The points $O,\ A$ and B are not collinear.) The point C has position vector $\boldsymbol{c}$ given by

$$\boldsymbol{c} = \alpha\boldsymbol{a} + \beta\boldsymbol{b},$$

where α and β are positive constants with $\alpha + \beta < 1$. The lines OA and BC meet at the point P with position vector $\boldsymbol{p}$ and the lines OB and AC meet at the point Q with position vector $\boldsymbol{q}$. Show that

$$\boldsymbol{p} = \frac{\alpha a}{1 - \beta}$$

and write down q in terms of $\alpha,\ \beta$ and b.

Show further that the point R with position vector r given by

$$\boldsymbol{r} = \frac{\alpha\boldsymbol{a} + \beta\boldsymbol{b}}{\alpha + \beta},$$

lies on the lines OC and AB.

The lines OB and PR intersect at the point S. Prove that $\frac{OQ}{BQ} = \frac{OS}{BS}$.

Exercise 11E Pg 246/247