SKILL #4: Reverse Chain Rule

There's certain more complicated expressions which look like the result of having applied the chain rule. I call this process '**consider then scale**':

1. **Consider some expression that will differentiate to something similar to it.**
2. **Differentiate, and adjust for any scale difference.**

$\int x(x^2+5)^3\,dx$	$\int \cos x \sin^2 x\ dx$	$\int \frac{2x}{x^2+1}\,dx$
The first x looks like it arose from differentiating the x^2	The $\cos x$ probably arose from differentiating the sin.	The $2x$ probably arose from differentiating the x^2.

✏ **Integration by Inspection/Reverse Chain Rule**: Use common sense to **consider some expression** that would differentiate to the expression given. Then **scale** appropriately.

Common patterns:

$$\int k\frac{f'(x)}{f(x)}dx \rightarrow \quad Try\ \ln|f(x)|$$

In words: "If the bottom of a fraction differentiates to give the top (forgetting scaling), try ln of the bottom".

$$\int kf'(x)[f(x)]^n \rightarrow \quad Try\ [f(x)]^{n+1}$$

$\int \frac{x^2}{x^3+1}\,dx$	$\int x\, e^{x^2+1}\,dx$

Quickfire

In your head!

$$\int \frac{4x^3}{x^4 - 1}\,dx =$$

$$\int \frac{\cos x}{\sin x + 2}\,dx =$$

$$\int \cos x \; e^{\sin x}\,dx =$$

$$\int \cos x\,(\sin x - 5)^7\,dx =$$

$$\int x^2(x^3 + 5)^7 =$$

Not in your head…

$$\int \frac{x}{(x^2 + 5)^3}\,dx =$$

Fro Tip: If there's as power around the whole denominator, DON'T use ln: reexpress the expression as a product. e.g. $x(x^2 + 5)^{-3}$

$\sin^n x \cos x$ vs $\sec^n x \tan x$

Notice when we differentiate $\sin^5 x$, then power decreases:

$$\frac{d}{dx}(\sin^5 x) =$$

However, when we differentiate $\sec^5 x$:

$$\frac{d}{dx}\left((\sec x)^5\right) =$$

Notice that the power of sec didn't go down. Keep this in mind when integrating.

$$\int \sec^4 x \tan x \;\, dx$$

Test Your Understanding

$$\int \sin x\,(\cos x + 1)^5\,dx$$

$$\int \frac{cosec^2 x}{(2+\cot x)^3}\,dx$$

$$\int \frac{\sec^2 2x}{\tan 2x + 1}\,dx$$

$$\int x(x^2+2)^3\,dx$$

$$\int 5\tan x \sec^2 x\;dx$$

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