Upper 6 Chapter 11

Integration

Chapter Overview

1. Integration
2. Integrals of the form $\boldsymbol{f'(ax+b)}$
3. Using Trigonometric Identities
4. The Reverse Chain Rule
5. Integration by Substitution
6. Integration by Parts
7. Integration Using Partial Fractions
8. Area Under a Curve
9. The Trapezium Rule
10. Parametric Equations
11. Differential Equations
12. Forming Differential Equations

8.1	**Know and use the Fundamental Theorem of Calculus**	**Integration as the reverse process of differentiation. Students should know that for indefinite integrals a constant of integration is required.**
8.2	**Integrate x^n (excluding $n=-1$) and related sums, differences and constant multiples.** Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples.	**For example, the ability to integrate expressions such as $\frac{1}{2}x^{2}-3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected.** x **Given f′(x) and a point on the curve, Students should be able to find an equation of the curve in the form $y=f(x)$.** To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x}, $\frac{1}{2x}$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
8.3	**Evaluate definite integrals; use a definite integral to find the area under a curve** and the area between two curves	Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines, or between two curves. This includes curves defined parametrically. For example, find the finite area bounded by the curve $y=6x-x^2$ and the line $y=2x$ Or find the finite area bounded by the curve $y=x^2-5x+6$ and the curve $y=4-x^2$.
8.4	Understand and use integration as the limit of a sum.	Recognise $\int_a^b f(x)\,dx=\lim_{\delta x\to 0}\sum_{x=a}^{b} f(x)\,\delta x$
8.5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.)	Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}dx=\ln f(x)+c$. The integral $\int \ln x\,dx$ is required Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.
8.3	**Evaluate definite integrals; use a definite integral to find the area under a curve** and the area between two curves	Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines, or between two curves. This includes curves defined parametrically. For example, find the finite area bounded

INTEGRATION

Integration is the reverse of differentiation. We use known derivatives to integrate.

The following are integrals that you should know:

SKILL #1: Integrating Standard Functions

There's certain results you should be able to integrate straight off, by just thinking about the opposite of differentiation.

y	$\int y\,dx$
x^n	
e^x	
$\frac{1}{x}$	
$\cos x$	
$\sin x$	
$\sec^2 x$	
$cosec\ x \cot x$	
$cosec^2 x$	
$\sec x \tan x$	

The $|x|$ has to do with problems when x is negative (when $\ln x$ is not defined)

Remember my memorisation trick of picturing sin above cos from C3, so that 'going down' is differentiating and 'going up' is integrating, and we change the sign if the wrong way round.

It's vital you remember this one.

Have a good stare at this slide before turning your paper over – let's see how many you remember...

Quickfire Questions (without cheating!)

$$\int \sec x \tan x \ dx = \square$$

$$\int \sin x \ dx = \square$$

$$\int cosec^2 \ x \ dx = \square$$

$$\int -\cos x \ dx = \square$$

Quickfire Questions (without cheating!)

$$\int \sec^2 x \ dx = \square$$

$$\int cosec\, x \cot x \ dx = \square$$

$$\int \frac{1}{x} \ dx = \square$$

$$\int -\sin x \ dx = \square$$

Quickfire Questions (without cheating!)

$$\int cosec^2 x \ dx = \square$$

$$\int \sin x \ dx = \square$$

$$\int \sec x \tan x \ dx = \square$$

$$\int \cos x \ dx = \square$$

Test Your Understanding

$$\int 2\cos x + \frac{3}{x} - \sqrt{x}\, dx =$$

$$\int \frac{\cos x}{\sin^2 x}\, dx = \int$$

Hint: What 'reciprocal' trig functions does this simplify to?

[Textbook] Given that $\int_a^{3a} \frac{2x+1}{x}\, dx = \ln 12$, find the exact value of a.

Important Notes:
We can simplify:

$$\frac{x+1}{x} \equiv \frac{x}{x} + \frac{1}{x} \equiv 1 + \frac{1}{x}$$

However it is **NOT** true that:

$$\frac{x}{x+1} \equiv \frac{x}{x} + \frac{x}{1}$$

In my experience students often fail to spot when they can split up a fraction to then integrate.

Exercise 11A Page 295