## C4 Integration

1 Integrate with respect to $x$
a $e^{x}$
b $4 \mathrm{e}^{x}$
c $\frac{1}{x}$
d $\frac{6}{x}$

2 Integrate with respect to $t$
a $2+3 \mathrm{e}^{t}$
b $t+t^{-1}$
c $t^{2}-\mathrm{e}^{t}$
d $9-2 t^{-1}$
e $\frac{7}{t}+\sqrt{t}$
f $\frac{1}{4} \mathrm{e}^{t}-\frac{1}{t}$
g $\frac{1}{3 t}+\frac{1}{t^{2}}$
h $\frac{2}{5 t}-\frac{3 \mathrm{e}^{t}}{7}$

3 Find
a $\int\left(5-\frac{3}{x}\right) d x$
b $\int\left(u^{-1}+u^{-2}\right) \mathrm{d} u$
c $\quad \int \frac{2 \mathrm{e}^{t}+1}{5} \mathrm{~d} t$
d $\int \frac{3 y+1}{y} \mathrm{~d} y$
e $\int\left(\frac{3}{4} \mathrm{e}^{t}+3 \sqrt{t}\right) \mathrm{d} t$
f $\int\left(x-\frac{1}{x}\right)^{2} \mathrm{~d} x$

4 The curve $y=\mathrm{f}(x)$ passes through the point $(1,-3)$.
Given that $\mathrm{f}^{\prime}(x)=\frac{(2 x-1)^{2}}{x}$, find an expression for $\mathrm{f}(x)$.
5 Evaluate
a $\int_{0}^{1}\left(\mathrm{e}^{x}+10\right) \mathrm{d} x$
b $\int_{2}^{5}\left(t+\frac{1}{t}\right) \mathrm{d} t$
c $\int_{1}^{4} \frac{5-x^{2}}{x} \mathrm{~d} x$
d $\int_{-2}^{-1} \frac{6 y+1}{3 y} \mathrm{~d} y$
e $\int_{-3}^{3}\left(e^{x}-x^{2}\right) d x$
f $\int_{2}^{3} \frac{4 r^{2}-3 r+6}{r^{2}} \mathrm{~d} r$
g $\int_{\ln 2}^{\ln 4}\left(7-\mathrm{e}^{u}\right) \mathrm{d} u$
h $\int_{6}^{10} r^{-\frac{1}{2}}\left(2 r^{\frac{1}{2}}+9 r^{-\frac{1}{2}}\right) \mathrm{d} r \quad \mathbf{i} \quad \int_{4}^{9}\left(\frac{1}{\sqrt{x}}+3 \mathrm{e}^{x}\right) \mathrm{d} x$

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The shaded region on the diagram is bounded by the curve $y=3+\mathrm{e}^{x}$, the coordinate axes and the line $x=2$. Show that the area of the shaded region is $\mathrm{e}^{2}+5$.

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The shaded region on the diagram is bounded by the curve $y=2 x+\frac{1}{x}$, the $x$-axis and the lines $x=1$ and $x=4$. Find the area of the shaded region in the form $a+b \ln 2$.

8 Find the exact area of the region enclosed by the given curve, the $x$-axis and the given ordinates. In each case, $y>0$ over the interval being considered.
a $y=4 x+2 \mathrm{e}^{x}, \quad x=0, \quad x=1$
b $y=1+\frac{3}{x}, \quad x=2, \quad x=4$
c $y=4-\frac{1}{x}, \quad x=-3, \quad x=-1$
d $y=2-\frac{1}{2} \mathrm{e}^{x}, \quad x=0, \quad x=\ln 2$
e $y=\mathrm{e}^{x}+\frac{5}{x}, \quad x=\frac{1}{2}, \quad x=2$
f $y=\frac{x^{3}-2}{x}, \quad x=2, \quad x=3$

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The diagram shows the curve with equation $y=9-\frac{7}{x}-2 x, x>0$.
a Find the coordinates of the points where the curve crosses the $x$-axis.
b Show that the area of the region bounded by the curve and the $x$-axis is $11 \frac{1}{4}-7 \ln \frac{7}{2}$.
10 a Sketch the curve $y=\mathrm{e}^{x}-a$ where $a$ is a constant and $a>1$.
Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equation of any asymptotes.
b Find, in terms of $a$, the area of the finite region bounded by the curve $y=\mathrm{e}^{x}-a$ and the coordinate axes.
c Given that the area of this region is $1+a$, show that $a=\mathrm{e}^{2}$.
11


The diagram shows the curve with equation $y=\mathrm{e}^{x}$. The point $P$ on the curve has $x$-coordinate 3 , and the tangent to the curve at $P$ intersects the $x$-axis at $Q$ and the $y$-axis at $R$.
a Find an equation of the tangent to the curve at $P$.
b Find the coordinates of the points $Q$ and $R$.
The shaded region is bounded by the curve, the tangent to the curve at $P$ and the $y$-axis.
c Find the exact area of the shaded region.
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\mathrm{f}(x) \equiv\left(\frac{3}{\sqrt{x}}-4\right)^{2}, x \in \mathbb{R}, x>0 .
$$

a Find the coordinates of the point where the curve $y=\mathrm{f}(x)$ meets the $x$-axis.
The finite region $R$ is bounded by the curve $y=\mathrm{f}(x)$, the line $x=1$ and the $x$-axis.
b Show that the area of $R$ is approximately 0.178

