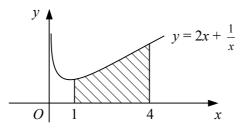
## INTEGRATION

**C4** 

1 Integrate with respect to x  $c = \frac{1}{r}$ d  $\frac{6}{r}$  $\mathbf{a} \mathbf{e}^x$ **b**  $4e^x$ 2 Integrate with respect to t **a**  $2 + 3e^{t}$  **b**  $t + t^{-1}$  **c**  $t^{2} - e^{t}$  **e**  $\frac{7}{t} + \sqrt{t}$  **f**  $\frac{1}{4}e^{t} - \frac{1}{t}$  **g**  $\frac{1}{3t} + \frac{1}{t^{2}}$ **d**  $9-2t^{-1}$ **h**  $\frac{2}{5t} - \frac{3e^{t}}{7}$ 3 Find **a**  $\int (5 - \frac{3}{x}) dx$  **b**  $\int (u^{-1} + u^{-2}) du$  **c**  $\int \frac{2e^{t} + 1}{5} dt$  **d**  $\int \frac{3y + 1}{y} dy$  **e**  $\int (\frac{3}{4}e^{t} + 3\sqrt{t}) dt$  **f**  $\int (x - \frac{1}{x})^{2} dx$ The curve y = f(x) passes through the point (1, -3). 4 Given that  $f'(x) = \frac{(2x-1)^2}{x}$ , find an expression for f(x). 5 Evaluate **a**  $\int_{0}^{1} (e^{x} + 10) dx$  **b**  $\int_{2}^{5} (t + \frac{1}{t}) dt$  **c**  $\int_{1}^{4} \frac{5 - x^{2}}{x} dx$ **d**  $\int_{-2}^{-1} \frac{6y+1}{3y} dy$  **e**  $\int_{-3}^{3} (e^x - x^2) dx$  **f**  $\int_{2}^{3} \frac{4r^2 - 3r + 6}{r^2} dr$ **g**  $\int_{\ln 2}^{\ln 4} (7 - e^u) \, du$  **h**  $\int_{6}^{10} r^{-\frac{1}{2}} (2r^{\frac{1}{2}} + 9r^{-\frac{1}{2}}) \, dr$  **i**  $\int_{4}^{9} (\frac{1}{\sqrt{x}} + 3e^x) \, dx$ 6  $y = 3 + e^x$ 

The shaded region on the diagram is bounded by the curve  $y = 3 + e^x$ , the coordinate axes and the line x = 2. Show that the area of the shaded region is  $e^2 + 5$ .

7



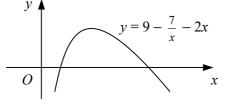
The shaded region on the diagram is bounded by the curve  $y = 2x + \frac{1}{x}$ , the x-axis and the lines x = 1 and x = 4. Find the area of the shaded region in the form  $a + b \ln 2$ .

## C4 INTEGRATION

8 Find the exact area of the region enclosed by the given curve, the x-axis and the given ordinates. In each case, y > 0 over the interval being considered.

**a**  $y = 4x + 2e^{x}$ , x = 0, x = 1 **b**  $y = 1 + \frac{3}{x}$ , x = 2, x = 4 **c**  $y = 4 - \frac{1}{x}$ , x = -3, x = -1 **d**  $y = 2 - \frac{1}{2}e^{x}$ , x = 0,  $x = \ln 2$  **e**  $y = e^{x} + \frac{5}{x}$ ,  $x = \frac{1}{2}$ , x = 2**f**  $y = \frac{x^{3} - 2}{x}$ , x = 2, x = 3

9



The diagram shows the curve with equation  $y = 9 - \frac{7}{x} - 2x$ , x > 0.

- **a** Find the coordinates of the points where the curve crosses the *x*-axis.
- **b** Show that the area of the region bounded by the curve and the x-axis is  $11\frac{1}{4} 7 \ln \frac{7}{2}$ .

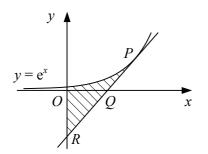
10 a Sketch the curve  $y = e^x - a$  where a is a constant and a > 1.

Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equation of any asymptotes.

- **b** Find, in terms of *a*, the area of the finite region bounded by the curve  $y = e^x a$  and the coordinate axes.
- **c** Given that the area of this region is 1 + a, show that  $a = e^2$ .

11

12



The diagram shows the curve with equation  $y = e^x$ . The point *P* on the curve has *x*-coordinate 3, and the tangent to the curve at *P* intersects the *x*-axis at *Q* and the *y*-axis at *R*.

**a** Find an equation of the tangent to the curve at *P*.

**b** Find the coordinates of the points Q and R.

The shaded region is bounded by the curve, the tangent to the curve at P and the y-axis.

c Find the exact area of the shaded region.

$$f(x) \equiv (\frac{3}{\sqrt{x}} - 4)^2, x \in \mathbb{R}, x > 0.$$

**a** Find the coordinates of the point where the curve y = f(x) meets the x-axis.

The finite region R is bounded by the curve y = f(x), the line x = 1 and the x-axis.

**b** Show that the area of *R* is approximately 0.178